REYNOLDS DECOMPOSITION OF TURBULENCE CONTAINING

SUPER-COHERENT STATES

Ronald Adrian Philip Sakievich

School for Engineering of Matter , School for Engineering of Matter,

Transport and Energy, Transport and Energy,

Arizona State University, Arizona State University,

Tempe, AZ 6106, USA Tempe, AZ 6106, USA

rjadrian@asu.edu psakievich@gmail.com

Yulia Peet

School for Engineering of Matter, Transport and Energy,

Arizona State University,

Tempe, AZ 6106, USA

ypeet@asu

ABSTRACT

A vexing problem occurs in certain flows when long, but finite, time-average estimates of the mean flow fail to exhibit the symmetry properties imposed by boundary conditions and physics. The mean field becomes suspect, making it difficult, or even incorrect to apply Reynolds decomposition. The problem occurs when the flow exhibits ‘super-coherent’ states, i.e. states of flow having coherence times much longer than the averaging times used in typical turbulence experiments. Turbulent Rayleigh-Benard convection (RBC) is one such flow, and it will be used here as an example to illustrate and explain this phenomenon. The study focuses on a turbulent RBC experiment (Fernandez, R. 2000) in a 6.3:1 (diameter: depth) aspect-ratio vertical cylinder that supplemented time averaging with true ensemble averaging to achieve almost zero mean flow. To obtain a three-dimensional time-varying picture of the mechanisms at work, the experiment is simulated by direct numerical simulation of the Boussinesq equations (Sakievich, et al. 2016). Three types of super-coherent states, associated with the symmetries of the flow, are found to bias the mean flow, unless steps are taken to sample each state with equal probability.

INTRODUCTION

Reynolds decomposition is fundamental in the analysis of turbulence, and measurement of the mean flow must be accurate to within a small fraction of the turbulent fluctuation intensity to properly define the turbulent fluctuating field. The rules for averaging the results of turbulent flow experiments, physical or numerical, are quite simple: infinite averages over time are unbiased if the flow is statistically stationary and ergodic; infinite averages over space are unbiased if the flow is statistically homogeneous in the averaging direction(s) and ergodic; and finite averages converge to the infinite averages with small random error if they are performed over many thousand integral scales (in time or space). Lastly, convergence should be checked by repeated experiments.

These rules suffice in most situations, if the averaging domains are large enough to achieve converged statistics, but not always. In certain flows they do not guarantee unbiased results because, even if convergence is suggested by smoothness of the average, it may be much slower may be much slower than expected, leading to physically incorrect results.

A more insidious problem with time averaging occurs when the flow locks into ‘states’ that do not possess all of the symmetry that the infinite time average must have. These seem to

result from state-space bifurcations into basins of attraction that trap the dynamics for very long (or perhaps infinite), times only occasionally (or perhaps never) allowing natural transitions from one basin to another. In these cases, averages over long but finite times may sample only one of the states, or sample one state over a much longer time than another, creating a bias. It is necessary to either take much longer time averages, many times not an option, or to stimulate the transitions. The latter can be accomplished by stopping the experiment and starting a new one, so as to achieve identical, independent experiments, yielding a finite ensemble of equi-probable experiments. This approach holds true to the definition of an ensemble average, and so long as each state is realized with equal frequency, it is shown to improve convergence to the true infinite time average considerably (Fernandes, 2000).

Two new methods are introduced to reduce bias to one state or another in a finite time experiment. One involves sensing the times spent in the different states, and the other defines an efficient way to stop and restart the identical experiments so as to achieve independent realizations whose initial conditions occupy the various basins of attraction with the correct frequency of occurrence.

Turbulent Rayleigh-Benard Convection

The ideal canonical form of RBC occurs in a horizontal layer of constant property fluid bounded by two infinitely wide horizontal plates, the warm bottom plate being heated uniformly and steadily, and the cool top plate being cooled in a similar manner to achieve either constant temperatures or constant mean heat flux. The controlling parameters are the Rayleigh number and the Prandtl number,

Ra=

Pr=

This flow has been studied extensively, albeit perforce using plates of finite width as characterized by the aspect ratio,

Gamma= width/height

Linear instability of this system occurs at Ra=xxxx, and it has the form of parallel, steady, two-dimensional roll-cells. With increasing Rayleigh number a sequence of more complicated finite amplitude laminar instabilities and transitions occurs (Busse, Busse et al., ???.), ending with steady, laminar hexagonal cells at about Ra= 50,000. Above Ra= 10^5 the flow becomes chaotic, and around Ra=10^6-10^7 it is usually considered to be turbulent. Studies of the turbulent state (Malkus, Busse, et. al., Goldstein and Chu, Aaron and Goldstein, Deardorff and Willis (1970), Fitzgerald) were generally performed in square or rectangular test sections whose aspect ratios ranging from xx.x to xx.x. Despite being fairly large, it was found in a number of experiments that non-zero mean flows existed in the test section, contrary to the zero-value expected for infinite aspect-ratio. As noted above this puzzling result made application of Reynolds decomposition problematic. A similar phenomenon also appeared in unsteady non-penetrative convection, a very close relative of RBC in which the cool upper plate is replaced by an insulating plate (Adrian, Boberg and Ferriera, 1986). Means flows were initially attributed to imperfections in the experiments, including insufficient aspect-ratio. But the careful experiments of Krishnamurty and Howard 19??) in square and annular test section also possessed mean flows, showing conclusively that non-zero mean flow in RBC derives from its fluid mechanics rather than experimental imperfections.

INSERT LOW ASPECT-RATIO

It is commonly observed that mean flow is prominent in unit aspect-ratio cubes and cylinders (Libchaber, et al. Bodenschatz, et al.; Emran, et al. 2015). Originally called the ‘wind of turbulence’, the mean flow is part of a large-scale circulation that sweeps across the upper and lower plates, Fig. 1 creating a boundary layer that is of considerable interest regarding heat transfer.

At first blush one is tempted to attribute LSC’s to low aspect ratio, but we find mean flow patterns in our HPC simulations of turbulent RBC in a wide circular cylinder having aspect-ratio gamma=6.3. The flat cylinder characteristically contains large-scale motions that are mean-square-periodic in the azimuthal-direction. In plan-view they look like a somewhat randomly centered hub of rising (falling) fluid from which ‘spokes’ emanate in radial directions, Figure 2. The spokes are convergence zones of warm (cool) fluid rising (falling) between horizontal, radially oriented, counter-rotating roll-cells. The most energetic roll-cells have azimuthal periodicity.

The initial azimuthal orientation of the pattern is random, and while it changes randomly, we have never seen the orientation sample all possible directions during the computational observation times. The coherence times for each mode, defined as the integral time scales of the time varying Fourier coefficient, have values much longer than one expects from the usual time scales. For example, we observed correlation over more than 20 eddy turn-over times. The failure to assume all orientations in a reasonable averaging time can be can be eliminated by averaging in the statistically homogeneous azimuthal direction.

Very slowly evolving coherent motions have been observed in experimental studies and numerical simulations of wide aspect-ratio, turbulent Rayleigh-B\'{e}nard convection. The time scales on which they evolve make it extremely difficult to achieve statistically-converged results during the finite run times of numerical simulations. In this paper, we present a novel procedure of manipulating the turbulent flow states and performing statistical averaging in a way that mitigates these problems and yields the flow statistics that are close to the results of experiments which, in turn, approach the infinite-time average.

Most theories of turbulence assume the flow is statistically stationary so that averages over infinite times converge to the ensemble average of an infinite number of random realizations. This makes the infinite-time average calculable in principle.

In experiments and numerical simulations the infinite-time average is unreachable, and time averages over finite times often fail to converge well. Supplemental spatial averages over regions of homogeneous statistics or supplemental ensemble averages over additional realizations are often invoked to improve convergence of the finite-time average.

**Extension to other flows**

The majority of the turbulent flows contain some kind of large-scale, seemingly chaotic turbulent motion, and in some flows these large-scale motions organize into very-large-scale motions that evolve on extremely large time scales compared to the viscous time scales of the smallest eddies~\cite{kim1999very,balakumar2007large}. One example is turbulent Rayleigh-B\'{e}nard convection (RBC) in a wide-aspect-ratio, cylindrical domain~\cite{sakievich2016large}. The slowly evolving coherent motions make it very difficult to use conventional time averaging procedures to obtain statistically-converged results over the finite-time of numerical simulations. Spatial averaging over homogeneous directions helps somewhat, as will be shown below, but it does not completely cure the problem. The purpose of this letter is to propose a new technique that accounts for the influence of multiple states in large-scale organization of coherent structures and

combines temporal and a specially-constructed ensemble averaging to significantly improve the statistical convergence in finite-time simulations of Rayleigh-B\'{e}nard convection.

RBC occurs when fluid between horizontal plates is heated from below and cooled from above. The unstable temperature stratification generates buoyancy forces within the fluid layer which then drive the flow. The Rayleigh number $Ra=\beta g \Delta T h^3/\alpha \nu$, (where $\beta$ is the coefficient of thermal expansion, $g$ is the gravitational constant, $\Delta T$ is the temperature difference between the two heated plates, $h$ is the plates' vertical separation, $\alpha$ is the thermal diffusivity and $\nu$ is the kinematic viscosity), is the primary dimensionless parameter and the Prandtl number $Pr=\nu/\alpha$ is often of less importance. A horizontal length scale ($L$) is also very important for determining the structure of the flow. The ratio of these two length scales is the aspect-ratio ($\Gamma=L/h$).

The majority of numerical and experimental studies have been performed in unit $\Gamma$ boxes and cylinders. The ``wind of turbulence'' concept is often used to describe the flow structure in these small $\Gamma$ domains. The ``wind of turbulence'' is characterized by a single roll-cell, or large-scale circulation (LSC), which spans the height and width of the cell, see figure~\ref{fig:1ar}. This roll-cell creates boundary layers along the side walls and thermally active top and bottom plates which are well described by the Prandtl-Blasius profiles according to the Grossmann and Lohse theory~\cite{grossmann2000scaling}. The core of these small $\Gamma$ cells is well-mixed and shows statistical In boxes the LSC may align with the side-walls, but in cylinders circular symmetry of the side-walls and verticality of the gravitational vector combine to imply that there can be no preferred horizontal direction, i.e. structures can align in any horizontal direction, and the infinite-time mean of any quantity must be independent of the azimuthal direction (azimuthal homogeneity). In particular, the LSC is allowed to flow in any direction. Experiments and numerical simulations show that the direction of the LSC (and presumably the azimuthal orientation of any flow pattern) drifts in time~\cite{brown2005reorientation,mishra2011dynamics}, so that over an infinite time all azimuthal orientations become equally probable, implying statistical homogeneity in the azimuthal direction and suggesting the azimuthal averaging as a means to accelerate statistical convergence to an infinite time-average in this flow. In horizontal RBC cells the anti-symmetry of the thermal boundary conditions on the horizontal surfaces also implies anti-symmetry of statistical means for quantities involving temperature or heat flux with respect to reflection about the horizontal mid-plane.

The anti-symmetry about the mid-plane is due to the vertical direction of the gravitational vector and the equal and opposite temperatures (with respect to the mean value) of the thermally active boundaries. There is nothing in the equations or boundary conditions to give preference to updrafts or down-drafts, so thermal plumes rise (fall) from the lower (upper) boundary with equal likelihood. Referring to figure~\ref{fig:1ar}, the updraft on the left hand side of the flow has an equal probability of being a down-draft over an infinite time. When the large-scale circulation is a single roll-cell $180\degree$ rotation about the central axis changes the updraft on the left to a down-draft. However, as $\Gamma$ is increased the flow's structure acquires a more complicated form than the relatively two dimensional ``wind of turbulence'' and azimuthal homogeneity can diverge from anti-symmetry in the vertical direction

%\section{\label{sec:level1}Additional states in larger aspect-ratio cylinders}

In our recent work we studied the large-scale structures in a 6.3 $\Gamma$ RBC cell via direct numerical simulation (DNS)~\cite{sakievich2016large}. This simulation was setup to mirror an experiment conducted by Fernandes~\cite{Fernandes}. After smoothing out the small-scales with a running time average we observed that the flow organized itself into a hub and spoke like pattern with an updraft in the central region of the cell, and 6 alternating up- and down-drafts near the outer wall. The hub in this pattern is the central thermal and the spokes are the vortex lines that form between drafts of opposing direction along the outer wall. A conceptual illustration of the observed pattern's thermal signature is provided in figure~\ref{fig:63ar}. Very similar patterns were seen in the numerical study by Bailon-Cuba \textit{et al}~\cite{bailon2010aspect}. The large-scale patterns in our recent work~\cite{sakievich2016large} and the work of Bailon-Cuba \textit{et al}~\cite{bailon2010aspect} showed no azimuthal drift or vertical reversal over at least 600 free fall time units ($t\_f=\sqrt{h/\beta g \Delta T}$) in numerical simulations. From this we can infer that the large-scale patterns in turbulent RBC are remarkably stable at large $\Gamma$.

**PHIL, Please look at the references to make sure I classified them correctly**

**References**

**Early RBC studies**

**Instabilities and transitions**

Busse, F. He, and J. A. Whitehead. "Instabilities of convection rolls in a high Prandtl number fluid." *Journal of Fluid Mechanics* 47.02 (1971): 305-320.

Busse, F. H.; Whitehead, J. A. 1974: Oscillatory and convective

instabilities in large Prandtl number convection. J. Fluid

Mech. 66, 67-69

Busse, F. H. 1978: Nonlinear properties of thermal convection.

Rep. Prog. Phys. 41, 1931-1967

Busse, F. H.; Riahi, N. 1980: Nonlinear

**Turbulence**

Malkus W V 1954

Deardorff, J. W.; Willis, G. E. 1967 a: Investigation of turbulent

thermal convection between horizontal plates. J. Fluid Mech.

28, 675-704

Deardorff, J. W.; Willis, G. E. 1967b: The free-convection temperature

profile. Q. J. Roy. Meteor. Soc. 73, 166-175

J. W. Deardorff, Journal of the Atmospheric Sciences 27,

1211 (1970).

A. M. Garon and R. J. Goldstein, Velocity and heat

transfer measurements in thermal convection, Phys.

Fluids 16, 1818-1825 (1973).

Chu, T. Y.; Goldstein, R. J. 1973: Turbulent convection in a

horizontal layer of water. J. Fluid Mech. 60, 141-159

Fitzjarrald, D. E. 1976: An experimental study of turbulent convection

in air. J. Fluid Mech. 73, 693-719

Grossmann, Siegfried, and Detlef Lohse. "Scaling in thermal convection: a unifying theory." *Journal of Fluid Mechanics* 407 (2000): 27-56.

Schumacher, Jörg, Janet D. Scheel, Dmitry Krasnov, Diego A. Donzis, Victor Yakhot, and Katepalli R. Sreenivasan. "Small-scale universality in fluid turbulence." *Proceedings of the National Academy of Sciences* 111, no. 30 (2014): 10961-10965.

Chu, T. Y.; Goldstein, R. J. 1973: Turbulent convection in

**Early observations of mean flows in large aspect ratio RBC**

Adrian Ferriera and Boberg

Yao thesis

Fitzjarrald, D. E. 1976: An experimental study of turbulent convection

in air. J. Fluid Mech. 73, 693-719

Krishnamurty R L and Howard, L. N.

Krishnamurti, Ruby, and Louis N. Howard. "Large-scale flow generation in turbulent convection." *Proceedings of the National Academy of Sciences* 78.4 (1981).

R. L. Fernandes, Ph.D. thesis, University of Illinois, Ur-

bana, IL (2001).

R. L. Fernandes and R. J. Adrian, Exp. Thermal Fluid

Science 26, 355 (2002).

J. Bailon-Cuba, M. S. Emran, and J. Schumacher, Jour-

nal of Fluid Mechanics 655, 152 (2010).

Xia, Ke-Qing, Chao Sun, and Yin-Har Cheung. "Large scale velocity structures in turbulent thermal convection with widely varying aspect ratio." *Proceedings of the 14th International Symposium on Applications of Laser Techniques to Fluid Mechanics*. 2008.

**Recent Reviews**

Bodenschatz E., Pesch W. & Ahlers G. 2000 Recent developments in Rayleigh-Bénard convection Annu. Rev.Fluid Mechanics 32, 709-778.

Ahlers, Guenter, Siegfried Grossmann, and Detlef Lohse. "Heat transfer and large scale dynamics in turbulent Rayleigh-Bénard convection." *Reviews of modern physics* 81.2 (2009): 503.

**Coherent structures-Small**

Zocchi, G., Moses, E. & Libchaber, A. 1990 Coherent

structures in turbulent convection, an experimental study.

Physica A 166, 387–407.

Puthenveettil, B. A. & Arakeri, J. 2005 Plume structure in

high Raleigh-number convection. Journal of Fluid Mechanics

542, 217–249.

Shelly, M. J. & Vinson, M. 1992 Coherent structures on

a boundary layer in Rayleigh-Benard turbulence. Nonlinearity

5, 323–351.

**Coherent Structures-Large**

Xi, Heng-Dong, Siu Lam, and Ke-Qing Xia. "From laminar plumes to organized flows: the onset of large-scale circulation in turbulent thermal convection." *Journal of Fluid Mechanics* 503 (2004): 47-56.

Emran, M. S., Schumacher, J. 2015 Large-scale mean patterns in turbulent convection. J. Fluid Mechanics 776, pp. 96-108.

Qui, X. L. & Tong, P. 2001 Large-scale structures in turbulent

thermal convection. Physical Review E 64 (036304).

Shishkina, O. & Wagner, C. 2007 Analysis of sheet-like

thermal plumes in turbulent Rayleigh-Benard convection.

Journal of Fluid Mechanics 599, 383–404.

Sakievich, P. J., Y. T. Peet, and R. J. Adrian. "Large-scale thermal motions of turbulent Rayleigh–Bénard convection in a wide aspect-ratio cylindrical domain." *International Journal of Heat and Fluid Flow* 61 (2016): 183-196.

**Coherence times Long**

[7] P. K. Mishra, A. De, M. K. Verma, and V. Eswaran,

Journal of Fluid Mechanics 668, 480 (2011).

Brown, E., Nikolaenko, A. & Ahlers, G. 2005 Reorientation

of the large-scale circulations in turbulent Rayleigh-Benard

convection. Physical Review Letters 95 (084503).

Fernandes, R. L. 2001 The spatial structure of turbulent

Rayleigh-Benard convection. PhD thesis, University of

Illinois at Urbana-Champaign, Urbana, IL.

Sakievich, P. J., Peet, Y. & Adrian R. J. 2016 Large-scale thermal motions of turbulent Rayleigh–Bénard convection in a wide aspect-ratio cylindrical domain. Int. J. Heat Fluid

**Other flows with very large coherent structures**

K. Kim and R. Adrian, Physics of Fluids 11, 417 (1999).

B. Balakumar and R. Adrian, Philosophical Transactions

of the Royal Society of London A: Mathematical, Physi-

cal and Engineering Sciences 365, 665 (2007).

**Computation**

Fischer, P. F., Lottes, J. W. & Kherkemeir, S. G. 2015

nek5000 web page, https://nek5000.mcs.anl.gov/.

Scheel, Janet D., Mohammad S. Emran, and Jörg Schumacher. "Resolving the fine-scale structure in turbulent Rayleigh–Bénard convection." *New Journal of Physics* 15.11 (2013): 113063.